Letters

Comments on "A Coordinate-Free Approach to Wave Reflection from a Uniaxially Anisotropic Medium"

FEDOR I. FEDOROV

I would like to point out that part of the contents of this paper[1] appear similar to sections 6, 17, 21, and 26 of [1], with some variation of notation. Section III of the above paper gives formulas which represent particular cases of general relations given in sections 19 and 23 of [1].

The coordinate-free (covariant) approach to the theory of electromagnetic waves was first proposed in 1952 and was later developed in several papers and three monographs in Russian, as indicated in the references.

Reply[2] by Hollis C. Chen[3]

I would like to thank Dr. Fedorov for bringing to our attention the existence of some Russian literature, previously unknown in the West.

After some search, it seems clear that these works were published in some obscure journals and by minor publishers not widely known even in the USSR. None of the references cited are available in United States and British libraries, but I have traced a copy of [1] to the U.S. Library of Congress.

Finally I might add that intial perusal of [1] indicates that the author does not appear to have treated the problem solved in my paper.

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[1] H. C. Chen, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 331–336, Apr 1983

[2] Manuscript received October 20, 1987

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Comments on "A Spectral-Domain Analysis of Periodically Nonuniform Microstrip Lines"

LEO J. RADEMACHER

Section 26 of Floquet's paper [3] has become known as Floquet's theorem. Its application to transmission line problems leads indeed to a fundamental system like eq. (2) in the paper[1] in question, but with several values of β allowed. (Other fundamental systems are also possible [3, ch. II] but are of little practical importance.) Usually the lowest possible value $\beta = \beta_0$ is chosen as the phase constant of the forward-traveling periodic wave and $\beta = -\beta_0$ as the phase constant of the backward-traveling periodic wave. The general solution is the sum of some forward- and some backward-traveling periodic waves. Periodic waves (also referred to as Floquet modes) are used here instead of the ordinary well-known waves which transport power independently of each other. The latter ("partial waves" in [4]) will be called physical waves throughout this comment. Both forward- and backward-traveling periodic waves consist of both forward- and backward-traveling space harmonics with the phase constant $\beta = \pm\beta_0 + k2\pi/\text{p}$, where the plus sign is valid for a forward and the minus sign for a backward-traveling periodic wave. If such whole numbers k_1, k_2 exist that $\beta = +\beta_0 + k_1 2\pi/\text{p} = -\beta_0 + k_2 2\pi/\text{p}$, then the respective space harmonic is part of both the forward- and the backward-traveling periodic wave. Without knowing the characteristic impedances of forward- and backward-traveling periodic waves beforehand, it is very difficult to split the space harmonics into their components belonging to the forward- or backward-traveling periodic wave, respectively. For this purpose the authors use a physical plausibility check [2, p. 55] demanding the power transported by periodic waves not to vary along the line.

Collin stresses the importance of distinguishing between periodic and physical ("partial") waves [4, sec. 9.3]. Both forward- and backward-traveling periodic waves consist of both forward- and backward-traveling physical waves. Although referencing Collin's standard textbook [4], the authors are totally unaware of the foundations described there. In the first place they rather unclearly introduce the periodic waves used throughout their paper. ("In contrast to the case of the uniform microstrip line, in the case of the periodically nonuniform microstrip line the functions ... are still periodic functions of the coordinate z.") But in order to obtain unique solutions, they then use the physical plausibility check already mentioned above—although totally inappropriate for the periodic waves under consideration. (As a matter of fact they demand them to be physical waves at the same time!)

Their results represent some obscure mixture of a forward- and a backward-traveling periodic wave, instead of a pure forward-traveling periodic wave, as the reader is led to believe. Because both forward- and backward-traveling periodic waves have the same phase constant, this parameter has been calculated correctly and is presented in the paper in question, but characteristic impedances have not been included. As the voltages and currents shown in Figs. 11–14 in the paper include a backward-traveling periodic wave component, the characteristic impedance does not equal the ratio of voltage to current [4, sec. 9.3, eq. (25)], as the reader is led to believe.

The method is an application of Jansen's work [5] which has the great advantage of including the effects of loss and finite strip thickness. Because of the dependence on the z coordinate, a much more complicated series expansion has to be used. The authors apply a very special field theoretic description with special-case expansion functions that must be matched to each type of nonuniformity, such as zigzag or sine-shape contours.

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[1] F. J Glandorf and I. Wolff, *IEEE Trans. Microwave Theory Tech*, vol. MTT-35, pp 336–343, Mar. 1987.

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Only lines symmetric with respect to the z axis are considered, and only waves with an even y component of the electric field strength with x-coordinate dependence—not for "brevity and clearness" [p. 337 of the paper in question] but "in order to keep the numerical effort as small as possible" [2, p. 31]. In the general case, four times as many coefficients would have to be included in the analysis. For the same reason of allowing numerical solvability, two further oversimplifying assumptions are made: loss is totally disregarded and the metallization thickness is assumed zero—thus getting rid of the great advantages of Jansen's method [5]. In consequence, no information about the attenuation versus frequency behavior is delivered by the "rigorous" field theoretic approach, although these data are very important in design. Absolutely no results are available in stopbands. (The attenuation may increase in stopbands by only a few, but also by a few thousand percent compared with the attenuation in neighboring passbands.)

Remarkably, the numerical results (given only for passbands) are claimed by the authors to be in good agreement with measurements only "over a wide frequency range" (p. 343) and no measured values are given near the second stopband. It should also be noted that only moderate nonuniformities ($w_{\min}/w_{\max}$ up to 3.5) have been considered. Despite the "considerable" (p. 343) numerical effort, the authors calculate only one out of the six parameters describing nonuniform lines (attenuation constant, phase constant, real and imaginary parts of the characteristic impedances for the forward- and backward-traveling periodic waves). Within stopbands no parameter can be calculated at all. For a design process the results are therefore totally insufficient.

Reply[2] by F. J. Glandorf and I. Wolff[3]

We totally disagree with the comments of L. J. Rademacher, which need some fundamental clarifications. In detail:

1) Being aware of all the fundamental properties of periodic waves, as explained below, we did not want to repeat the fundamental literature and introduced Floquet's theorem as e.g. has been done by Collin [4, sec. 9.6, p. 388] and not "rather unclearly" as claimed by Rademacher.

2) Principally the general solution for a periodic structure of *finite length* is a sum of a forward- and a backward-traveling periodic wave. In the case of an infinitely long periodic waveguide, however, forward- and backward-traveling periodic waves can exist independently of each other. Only electromagnetic fields which represent a pure traveling periodic wave on the infinitely long guiding structure are considered in our paper. These periodic waves are the only physical solutions for the electromagnetic fields which can exist on the guiding structure under consideration [4, p. 388].

3) Collin indeed stresses the importance of distinguishing between periodic waves and space harmonics. [4, p. 388] and he clearly shows that only the periodic waves fulfill the boundary conditions on the periodic guiding structure, whereas the space harmonics cannot exist independently on their own, and therefore by no means are physical solutions.

4) In section 9.3, Collin [4] uses the term "partial waves" for electromagnetic waves existing on uniform connection lines between equidistant discontinuities, thereby forming a periodic structure. The "partial waves" are different from the above-mentioned space harmonics and they cannot be used in the context discussed in our paper. Their application requires an exact knowledge of the transmission characteristics of one subsection of the periodic guiding structure. The partial waves indeed are physical waves but they only exist on the homogenous connection lines.

5) Rademacher claims that for two integers k_1 and k_2 and

$$\beta = \beta_0 + k_1 \frac{2\pi}{p} = -\beta_0 + k_2 \frac{2\pi}{p} \tag{1}$$

a coupling between forward- and backward-traveling periodic waves occurs and that therefore no pure forward-traveling wave can be analyzed by the presented method without knowing *a priori* the characteristic impedances. In his arguments Rademacher overlooked that solving his conditional equation (1) leads to $\beta_0 \cdot p = (k_2 - k_1)\pi = k\pi$, where k is again an integer. This condition is only fulfilled at the boundaries between stopbands and passbands, e.g. [4, p. 387, fig. 9.9] or [6, p. 319, fig. 7.5.3]. The field distribution in this case is a pure standing wave [4, sec. 9.5, p. 387]. Even in this case the field solution can be described by the infinite sum of space harmonics as used in our paper (eq. (3)). Of course, in this case the solution for the backward-traveling wave ($\beta_0 < 0$) is identical with the solution of the forward-traveling wave ($\beta_0 > 0$) because both are standing waves and this leads to Rademacher's conditional equation (1). For all other values of β_0 the argument of Rademacher does not apply and need not be discussed.

6) What Rademacher calls a "plausibility check" is the fundamental physical condition called Poynting's law [4, p. 10]. In the case of a lossless guiding structure, eq. (25) of [4] leads to the relation

$$Re\left\{ \oint\!\!\oint_S (\vec{E} \times \vec{H}) \vec{n}\, dA \right\} = 0 \tag{2}$$

which is identical to the condition used in [2] that the power transported through the cross section of the periodic guiding structure be independent of the z coordinate along the line. Condition (2) must be applied to the total electromagnetic field, i.e., the fields of the periodic waves, and not to the different space harmonics separately (compare also [4, sec. 9.6, p. 388]).

7) Insofar the solutions presented in our paper by no means are mixtures of forward- and backward-traveling periodic waves. They are derived from a pure forward-traveling periodic wave. This also can clearly be seen from Figs. 11 to 14 in our paper. In case of two periodic waves traveling in opposite directions, the voltages and currents on the lines must exhibit an additional periodic dependence on the z coordinate with a periodicity of $\beta/2\pi$. This cannot be found in our results, as is very obvious from e.g. the results for $\beta_N = 0.1$. Indeed, the characteristic impedances of the forward-traveling periodic wave (excluding the discussion of the definition of characteristic impedances on microstrip lines [7]) can be calculated from the voltages and currents shown in Figs. 11 to 14, or from the transported power and the currents or voltages. These characteristic impedances are not identical to the impedances obtained by using formulas for the characteristic impedance of uniform microstrip lines and inserting $w(z)$!

8) Rademacher apparently was not aware also that the method of Jansen [5], which was the basis of our work, uses a field analysis method for microstrip structures with zero metallization thickness and no losses. Although losses and finite strip thickness are mentioned in the title of Jansen's paper, in the first sentence of the abstract of his paper he points out that with his optimized rigorous hybrid mode solutions, line characteristics (ϵ_{eff}, Z_0) are calculated on the basis of electromagnetic fields for zero metalli-

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zation thickness. The influence of the metallization thickness and the losses are introduced into the theory on the basis of these fields by approximate methods. Hence we did not eliminate the advantages of Jansen's method but instead benefited from his rigorous field formulations by using the same physical idealizations. As has been mentioned by Jansen in the paper referenced by Rademacher [5, p. 77]: "in view of the small percentage influence of t (metallization thickness) except for impracticably small strip widths (and spacings) the excess work involved in a rigorous solution of the finite thickness strip boundary value problem can surely not be justified for design purpose." As a result up to now (even nine years after the publication of Jansen's paper) no spectral-domain calculation taking into account finite metallization thickness is known to the authors from the literature.

In consequence of the assumed lossless guiding structure, the result, as is well known from the fundamental literature, e.g. [4], can be only that no wave propagation is possible in stopbands, in agreement with our results.

9) The measurements presented in our paper are over a frequency range from 1 GHz to 12 GHz, which is the frequency range of the HP 8410 network analyzer which was available for measurements. More measurement results, which also include results near the second stopband, can be found in [2].

10) L. Rademacher is right if he mentions that the numerical effort of the method is high, but on the other hand the method presented for the first time gives insight into the electromagnetic field distribution and the current density distribution of a complicated microwave structure. Additionally it is the first known rigorous solution for periodically nonuniform microstrip lines on which (as we hope) further research work can be based.

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